Lean burning below the flammability limit

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Introduction

The idea of burning at small concentration has been explored early in the past [1]. We investigate here the device in Fig. 1, where fresh gas is preheated in a counterflow configuration.

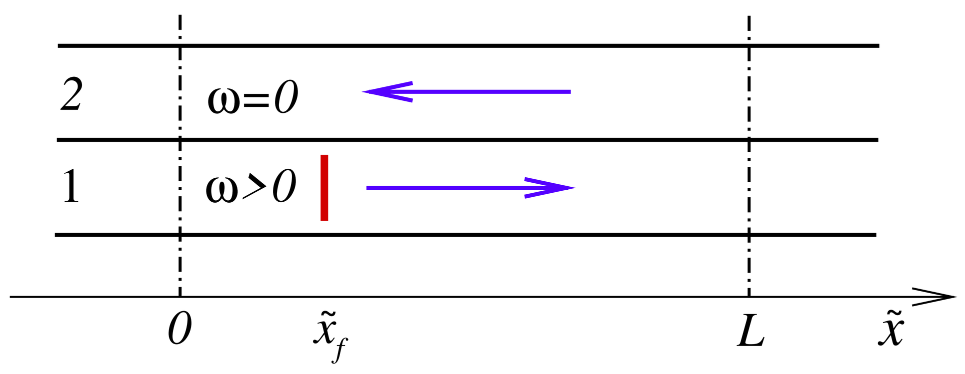

Figure 1: Counter-flow configuration

As the purpose of present work is to explicitly show the possibility of combustion for extremely lean mixtures, the classical single-step chemistry results not very convenient. Instead, we implement the two-step chain-branching kinetic model [2].

$$\begin{aligned} F + Z &\rightarrow 2Z && :\Omega_B = A_B \frac{\rho F}{W_F}\frac{\rho Z}{W_Z}\exp\left(\frac{-E}{R_g T}\right) \\ Z + M &\rightarrow P + M + Q && :\Omega_C = A_C \frac{\rho Z}{W_Z}\frac{\rho}{W} \end{aligned} \quad (1)$$

This mechanism defines a crossover temperature T_c below the flames extinguishes. The crossover temperature is given by the explicit expression $\Omega_B = \beta^2 \Omega_C$ where $\beta = \frac{E}{R_g T_b}\frac{T_b - T_u}{T_b}$ is the Zeldovich number.

Formulation

Equations nondimensionalized with the characteristic length $l_c = \sqrt{W\mathcal{D}_T/(\rho A_C)}$ and $u_c = \sqrt{\rho A_c \mathcal{D}_T/W}$ take the following form:

$$m\frac{\partial F}{\partial x} = \frac{1}{Le_F}\frac{\partial^2 F}{\partial x^2} - F\,Z\,\omega \quad (2)$$

$$m\frac{\partial Z}{\partial x} = \frac{1}{Le_Z}\frac{\partial^2 Z}{\partial x^2} + F\,Z\,\omega - Z \quad (3)$$

$$m\frac{\partial \theta_1}{\partial x} = \frac{\partial^2 \theta_1}{\partial x^2} + q\,Z \qquad -b\,\sigma(x/l)(\theta_1 - \theta_2) \quad (4)$$

$$m\frac{\partial \theta_2}{\partial x} = \frac{\partial^2 \theta_2}{\partial x^2} \qquad +b\,\sigma(x/l)(\theta_1 - \theta_2) \quad (5)$$

with

$$\omega = \beta^2 \exp\left(\beta\,(\theta_1 - 1)\,/\,(1 + \gamma\,(\theta_1 - 1))\right)$$

The choice of l_c as characteristic length comes from the will of burning in cases where no planar flame exists (and therefor we can not use the planar flame width as characteristic length). The σ function is

$$\sigma(x/l) = \begin{cases} 1, & \text{if } \ 0 < x/l < 1 \\ 0, & \text{if } \ x/l < 0 \text{ or } 1 < x/l \end{cases}$$

and b is the heat exchange rate. It is well-known that the dimensionless heat of reaction parameter $q = QF_0/c_p(T_c - T_0)W_F$ can not decrease below a value slightly lower than 1 for planar flames [3] . In particular, for the high activation energy limit $\beta \gg 1$, $q = 1$ at the flammability limit.

Equations (2)-(4) can be solved analytically in the case $\beta \gg 1$. This means that the chain-branching reaction rate ω is reduced to a very thin layer. Considering only the outer solution, the equations become linear and can be solved analytically

Solutions

The explicit solutions for Eqs. (2) and (3) are as follow:

$$F = \begin{cases} 1 - e^{Le_F\, m\,(x - x_f)}, & x < x_f \\ 0, & x > x_f \end{cases} \quad (6)$$

$$Z = \begin{cases} Z_f e^{a_m (x - x_f)}, & x < x_f, \\ Z_f e^{a_p (x - x_f)}, & x > x_f \end{cases} \quad (7)$$

with

$$a_m = \frac{Le_Z\, m + \sqrt{Le_Z^2 m^2 + 4Le_Z}}{2}, \quad a_p = \frac{Le_Z\, m - \sqrt{Le_Z^2 m^2 + 4Le_Z}}{2}$$

$$Z_f = \frac{Le_Z\, m}{\sqrt{Le_Z^2 m^2 + 4Le_Z}}$$

The solution of the temperature profile does not have a solution that can be easily computed. From the set up presented it can be extracted 8 boundary conditions that allow us to solve the equation by using mathematical tools such as MAPLE. The ninth condition provides a solvability condition of the form:

$$\mathcal{F}\left(x_f; m, q, l, b, Le_Z\right) = 0 \quad (8)$$

The explicit form of the function $\mathcal{F}$ and the solution of the temperature for both channels are expressions too lengthy to be written explicitly and does not reveal much about the nature of the solution. Typical examples of $\mathcal{F}$ function are shown in Fig. 2, where one can see the possible cases of this condition. Note that below a critical value of q there is no solution.

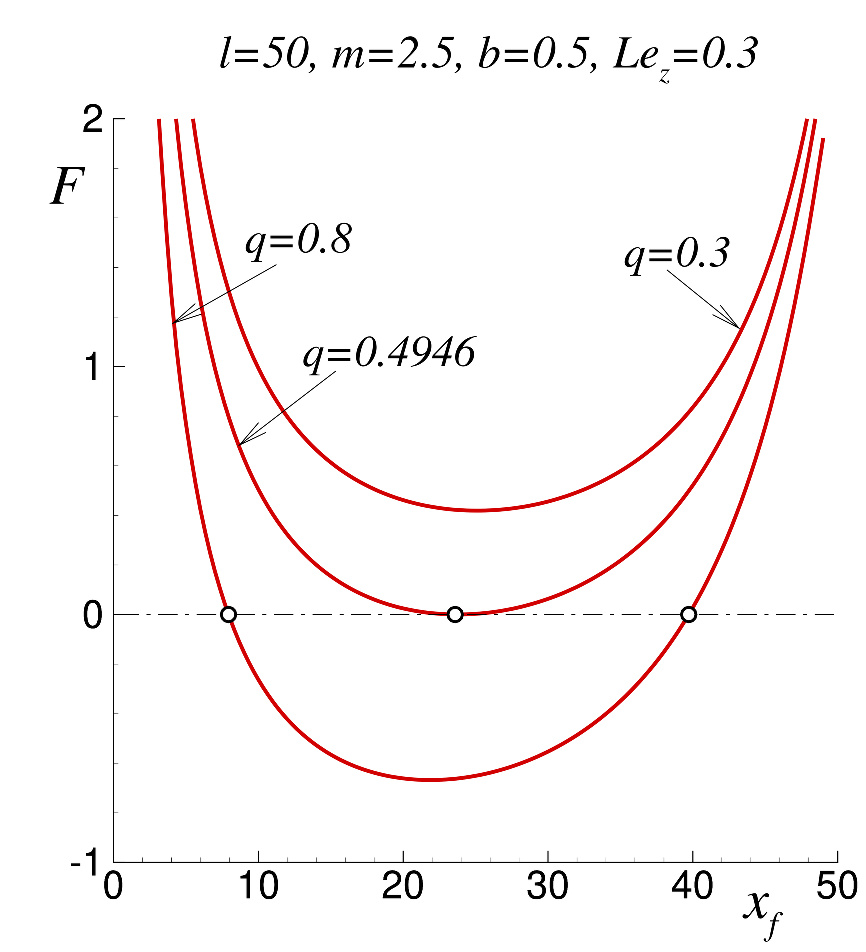

Figure 2: Typical examples of the $\mathcal{F}$ function

An example of the solutions that can arise from each set of parameters is presented below in Fig. 3. This case corresponds with two roots of the function $\mathcal{F}$

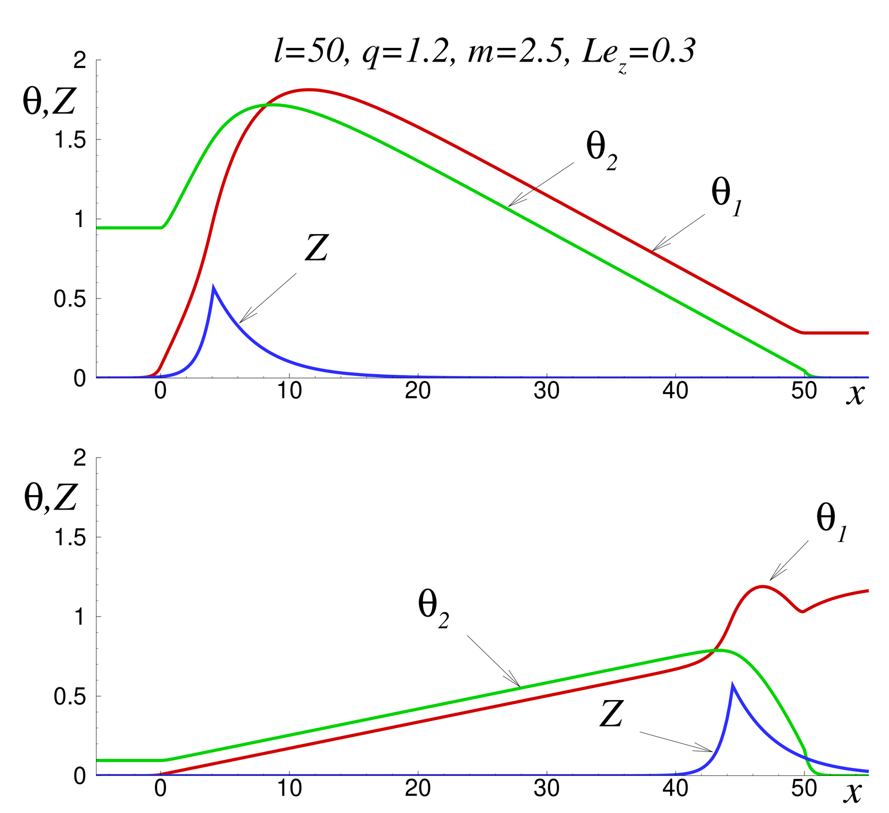

Figure 3: Distribution of the temperature and radical concentration

Fig. 4 shows the range of flames solutions as a function of the dimensionless flow rate m and the flame position x_f with the flame position being defined as $\theta_1 = 1$. The solutions shown in Fig. 3 correspond with the white dots in Fig. 4.

It can be seen that the range of velocities reduces as q is reduced. The most important point is the observation of flames below the flammability limit.

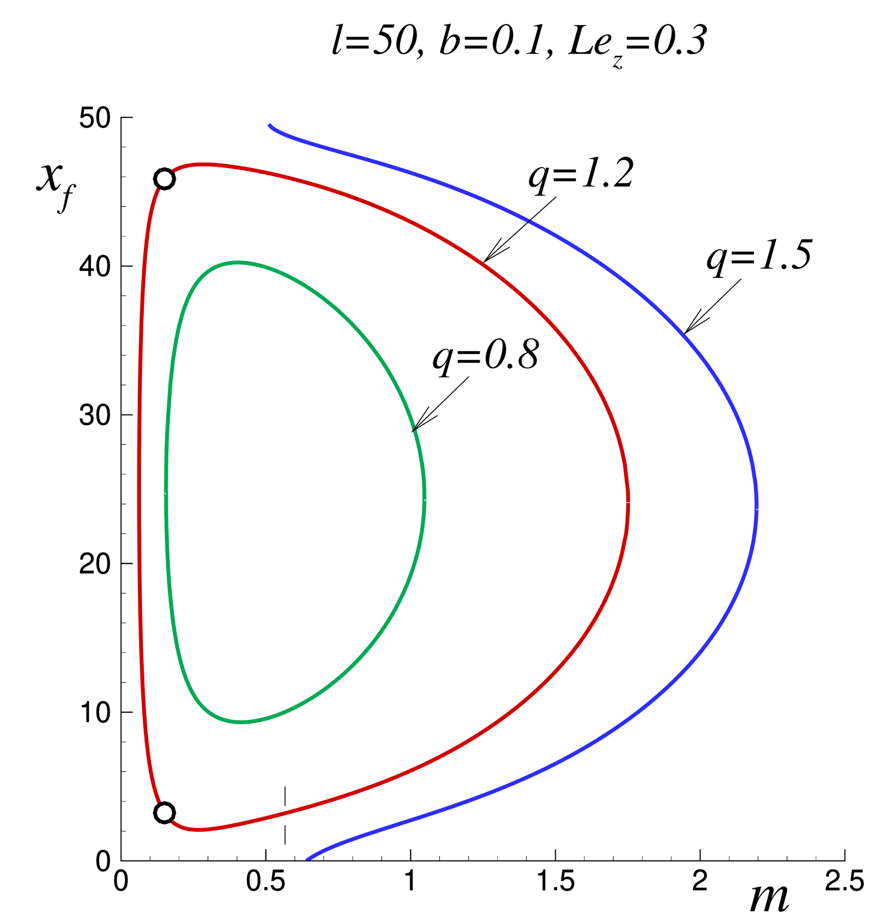

Figure 4: Range of velocities with possible solutions

As a validation of this analytical study we compare in Fig. 5 the obtained solution to the direct solutions of Eqs. (2)-(5) with a standard numerical code for different values of β. In the numerical solution, the position x_f is taken where Z peaks. In the case of $\beta = 10$ the small triangles show the points where $\theta_1 = 1$ when possible.

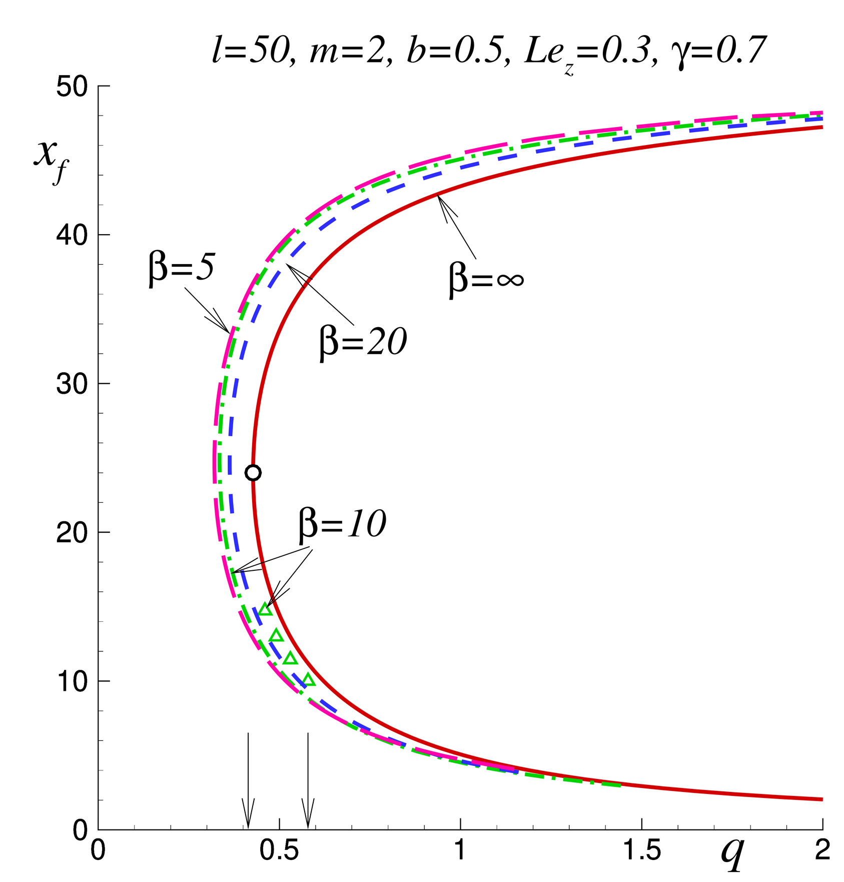

Figure 5: Comparison of the numerical and analytical solution

It can be seen from Fig. 5 that the analytical can be a good approximation of the numerical cases.

Conclusions

It has been shown that a self sustain combustion process can occur for lean mixtures under the flammability limits and the range of equivalence where combustion can be increased. The analytical analysis should not be underestimated and can be used in a future work to validate numerical solutions for a more complex and real like problem.

References

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